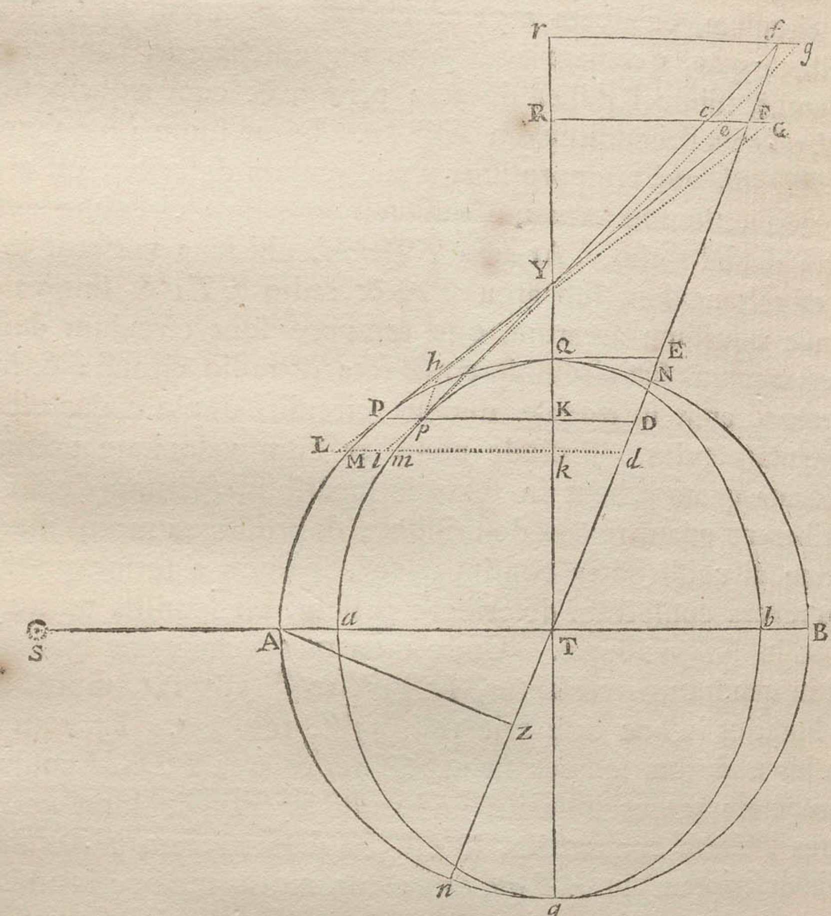
PROPOSITIO XXXI. PROBLEMA XII.

Invenire motum horarium nodorum lunæ in orbe elliptico.

Deſignet $Qpmaq$ ellipſin, axe majore Qq, minore ab deſcriptam, $QAqB$ circulum circumſcriptum, T terram in utriuſque centro communi, S ſolem, p lunam in ellipſi motam, & pm arcum quem data temporis particula quam minima deſcribit, N & n nodos



linea Nn junctos, pK & mk perpendicula in axem Qq demiſſa & hinc inde producta, donec occurrant circulo in P & M, & lineæ nodorum in D & d. Et ſi luna, radio ad terram ducto, aream deſcribat tempori proportionalem, erit motus horarius nodi in ellipſi ut area $pDdm$ & AZq conjunctim.

Nam ſi PF tangat circulum in P, & producta occurrat TN in F, & pf tangat ellipſin in p & producta occurrat eidem TN in f, conveniant autem hæ tangentes in axe TQ ad Υ; & ſi ML deſignet ſpatium quod luna in circulo revolvens, interea dum deſcribit arcum PM, urgente & impellente vi prædicta $3IT$, ſeu $3PK$ motu tranſverſo deſcribere poſſet, & ml deſignet ſpatium quod luna in ellipſi revolvens eodem tempore, urgente etiam vi $3IT$ ſeu $3PK$, deſcribere poſſet; & producantur LP & lp donec occurrant plano eclipticæ in G & g; & jungantur FG & fg, quarum FG producta ſecet pf, pg & TQ in c, e & R reſpective, & fg producta ſecet TQ in r. Quoniam vis $3IT$ ſeu $3PK$ in circulo eſt ad vim $3IT$ ſeu $3pK$ in ellipſi, ut PK ad pK, ſeu AT ad aT; erit ſpatium ML vi priore genitum, ad ſpatium ml vi poſteriore genitum, ut PK ad pK, id eſt, ob ſimiles figuras $P\Upsilon Kp$ & $F\Upsilon Rc$, ut FR ad cR. Eſt autem ML ad FG (ob ſimilia triangula PLM, PGF) ut PL ad PG, hoc eſt (ob parallelas Lk, PK, GR) ut pl ad pe, id eſt (ob ſimilia triangula plm, cpe) ut lm ad ce; & inverſe ut LM eſt ad lm, ſeu FR ad cR, ita eſt FG ad ce. Et propterea ſi fg eſſet ad ce ut $f\Upsilon$ ad $c\Upsilon$, id eſt, ut fr ad cR (hoc eſt, ut fr ad FR & FR ad cR conjunctim, id eſt, ut fT ad FT & FG ad ce conjunctim) quoniam ratio FG ad ce utrinque ablata relinquit rationes fg ad FG & fT ad FT, foret fg ad FG ut fT ad FT; atque ideo anguli, quos FG & fg ſubtenderent ad terram T, æquarentur inter ſe. Sed anguli illi (per ea quæ in præcedente propoſitione expoſuimus) ſunt motus nodorum, quo tempore luna in circulo arcum PM, in ellipſi arcum pm percurrit: & propterea motus nodorum in circulo & ellipſi æquarentur inter ſe. Hæc ita ſe haberent, ſi modo fg eſſet ad ce ut $f\Upsilon$ ad $c\Upsilon$, id eſt, ſi fg æqualis eſſet $\frac{ce \times f\Upsilon}{c\Upsilon}$. Verum ob ſimilia triangula fgp, cep, eſt fg ad ce ut fp ad cp; ideoque fg æqualis eſt $\frac{ce \times fp}{cp}$; & propterea angulus, quem fg revera ſubtendit, eſt ad angulum priorem, quem FG ſubtendit, hoc eſt, motus nodorum in ellipſi ad motum nodorum in circulo, ut hæc fg ſeu $\frac{ce \times fp}{cp}$ ad priorem fg ſeu $\frac{ce \times f\Upsilon}{c\Upsilon}$, id eſt, ut $fp \times c\Upsilon$ ad $f\Upsilon \times cp$, ſeu fp ad $f\Upsilon$ &